Probability & Geometry Answers

1. We have right triangle with the area=4*5/2=10. Consider the line y<x. All the points which satisfy this equation (are below the line y=x) and lie in the triangular region obviously will have x more than y, which is exactly what we want (as x>y --> x-y>0).

   The probability that the point will be from this region is: Area of this region/Area of the triangle.

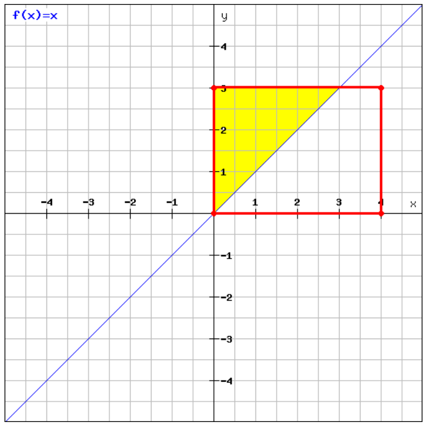
   Favorable region is also right triangle with vertexes at (0,0) (4,0) and (4,4). As y=x intersects the side of our original triangle at the point (4,4). You''ll see it easily if you draw it. So favorable are=4*4/2=8.

   P=8/10=4/5

   Answer: E.

2. See the diagram in the right. Now, rectangle R has an area of 3*4=12. All point that has y-coordinate greater than x-coordinate lie above the line $y = x$, so in yellow triangle, which has an area of 1/2*3*3=4.5. So, the probability equals to favorable outcomes/total=yellow triangle/rectangle R=4.5/12=3/8.

   Answer: C.



3. In order the area of a square to be more than 1, its side must be more than 1, or the perimeter must be more than 4. Which means that the longer piece must be more than 4. Look at the diagram below:

   If the wire will be cut anywhere at the red region then the rest of the wire (longer piece) will be more than 4 meter long. The probability of that is 2/5 (2 red pieces out of 5).

   Answer: E.

4. The area of equilateral triangle is $area = side^2 * \frac{\sqrt{3}}{4}$.

   For equilateral triangle the radius of the circumscribed circle is $R = side * \frac{\sqrt{3}}{3}$, thus the area of that circle is $\pi R^2 = \frac{\pi * side^2}{3}$.

   P = (the area of triangle)/(area of circle) = $\frac{3\sqrt{3}}{4\pi}$.

   Answer: C.

5. The area of the circle is $\pi r^2 = 100\pi$;
   The area of the square is $side^2 = 900$.

   The probability of hitting the circle is therefore $\frac{100\pi}{900} = \frac{\pi}{9}$ and the probability of missing the circle is $1 - \frac{\pi}{9} = \frac{9-\pi}{9}$.

   The probability of hitting the circle on at least one of the two attempts = 1 - {the probability of missing on both of the two attempts} =

   $1 - \frac{9-\pi}{9} * \frac{9-\pi}{9} = \frac{81-(9-\pi)^2}{81} = \frac{81-(81-18\pi+\pi^2)}{81} = \frac{18\pi-\pi^2}{81}$.

   Answer: E.

6. Given: $circumference = 4\sqrt{\pi\sqrt{3}}$ and $P(out) = \frac{3}{4}$

   Now, as the probability of the grain of sand landing on the portion of the base outside the triangle is 3/4 then the the portion of the base (circle) outside the triangle must be 3/4 of the are of the base and the triangle itself 1/4 of the are of the base.

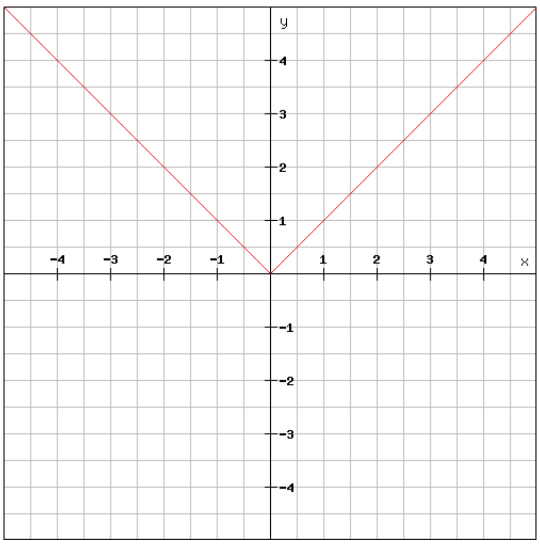
   Next: $circumference = 4\sqrt{\pi\sqrt{3}} = 2\pi r$ --> square both sides --> $16\pi\sqrt{3} = 4\pi^2 r^2$ --> $4\sqrt{3} = \pi r^2$ --> $area_{base} = \pi r^2 = 4\sqrt{3}$;

   The area of the equilateral triangle is 1/4 of the base: $area_{equilateral} = \frac{1}{4} * 4\sqrt{3} = \sqrt{3}$ --> also the ares of the equilateral triangle is $area_{equilateral} = a^2 * \frac{\sqrt{3}}{4}$, where a is the length of a side --

> $area_{equilateral}=a^2*\frac{\sqrt{3}}{4}=\sqrt{3}$ --> $a=2$.

Answer: E.

7. Below is given graph of y=|x|:



All points which satisfy $y\geq|x|$ condition lie above that graph. You can see that portion of the plane which is above the graph is 1/4.

Answer: E.

8. Since the probability of the marker landing on the portion of the base inside the triangle is $\frac{\sqrt{3}}{4}$ then the portion of the base (circle) inside the triangle must be $\frac{\sqrt{3}}{4}$ of the area of the base.

Next: $circumference=20\pi=2\pi r$ --> $r=10$ --> $area_{base}=\pi r^2=100\pi$;

The area of the equilateral triangle is $\frac{\sqrt{3}}{4}$ of the base: $area_{equilateral}=\frac{\sqrt{3}}{4}*100\pi$ --> also the ares of the equilateral triangle is $area_{equilateral}=a^2*\frac{\sqrt{3}}{4}$, where a is the length of a side --> $area_{equilateral}=a^2*\frac{\sqrt{3}}{4}=\frac{\sqrt{3}}{4}*100\pi$ --> $a=10\sqrt{\pi}$.

Answer: C.

9. The car ends within a half mile of the sign indicating 2 1/2 miles means that the car will end in one mile interval, between the signs indicating 2 and 3 miles.

   Now, it doesn't matter where the car starts or what distance it travels, the probability will be P=(favorable outcome)/(total # of outcomes)=1/3 (as the car starts at random point end travels some distance afterwards we can consider its end point as the point where he randomly appeared, so the probability that the car appeared within 1 mile interval out of total 3 miles will be 1/3). Answer is C.

10. First note that the square we have is centered at the origin, has the length of the sides equal to 2 and the area equal to 4.

    $x^2+y^2=1$ is an equation of a circle also centered at the origin, with radius 1 and the $area=\pi r^2=\pi$.

    We are told that the point is IN the square and want to calculate the probability that it's outside the circle ($x+y^2>1$ means that the point is outside the given circle).

    P=Favorable outcome/Total number of possible outcomes.

    Favorable outcome is the area between the circle and the square= $4-\pi$
    Total number of possible outcomes is the area of the square (as given that the point is in the square) =4

    $$P=\frac{4-\pi}{4}=1-\frac{\pi}{4}$$

    Answer: A.

11. A searchlight makes 1 revolution in 20 seconds. Consider the diagram below:

    A man randomly appears at some point M. Now, if beam of light is somewhere in the dark quarter, then the beam will need less than 5 seconds to reach a man and if beam of light is somewhere in the white 3 quarters then it'll need more than 5 seconds to reach a man (so he'll be in the dark more than 5 seconds).
    So P=3/4. Answer is E.